Oxford Cambridge and RSA

AS Level Further Mathematics A

Y534/01 Discrete Mathematics

Thursday 17 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae AS Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1 Some jars need to be packed into small crates.

There are 17 small jars, 7 medium jars and 3 large jars to be packed.

- A medium jar takes up the same space as four small jars.
- A large jar takes up the same space as nine small jars.

Each crate can hold:

- at most 12 small jars,
- or at most 3 medium jars,
- or at most 1 large jar (and 3 small jars),
- or a mixture of jars of different sizes.

(i) One strategy is to fill as many crates as possible with small jars first, then continue using the medium jars and finally the large jars.

Show that this method will use seven crates. **[2]**

The jars can be packed using fewer than seven crates.

(ii) The jars are to be packed in the minimum number of crates possible.

- Describe how the jars can be packed in the minimum number of crates.
- Explain how you know that this is the minimum number of crates. **[3]**

Some other numbers of the small, medium and large jars need to be packed into boxes.

The number of jars that a box can hold is the same as for a crate, except that

- a box cannot hold 3 medium jars.

(iii) Describe a packing strategy that will minimise the number of boxes needed. **[1]**

i. S: small m: medium L: large

crate	contents
1	12S
2	5S+1m
3	3m
4	3m
5	1L
6	1L
7	1L

$\Rightarrow$ 7 crates used

ii. Minimum no. crates ⇒ mininum/no spare capacity

crate	1	2	3	4	5	6
contents	1L+3S	1L+3S	1L+3S	3m	3m	1m+8s

$\therefore$ 6 is the minimum, as all crates are full

$[17+(7\times4)+(3\times9)]\div12=6$

iii. • put each L in a box

• put 2m in boxes until no more pairs remain

• put remaining m in another box

• fill spaces with S jars

2 Mo eats exactly 6 doughnuts in 4 days.

(i) What does the pigeonhole principle tell you about the number of doughnuts Mo eats in a day? **[1]**

Mo eats exactly 6 doughnuts in 4 days, eating at least 1 doughnut each day.

(ii) Show that there must be either two consecutive days or three consecutive days on which Mo eats a total of exactly 4 doughnuts. **[3]**

Mo eats exactly 3 identical jam doughnuts and exactly 3 identical iced doughnuts over the 4 days.

The number of jam doughnuts eaten on the four days is recorded as a list, for example 1, 0, 2, 0. The number of iced doughnuts eaten is not recorded.

(iii) Show that 20 different such lists are possible. **[3]**

i. there must be at least one day when Mo eats at least 2 doughnuts.

ii. must eat 1 each day $\Rightarrow$ max. on any day = 3

any day next to the 3 must have 1 eaten

$3+1=4$

OR 2 eaten on 2 days, 1 on 2 days.

if 2 eaten on adjacent days, $2+2=4$

Otherwise, 2 eaten on at least one end of the 4 days, & the other 3 total 4

order: 1113 1131 1311 3111

2211 1221 1122

2112 1212 2121

iii. $\frac{4!}{2!} = 12 \leftarrow$ no. arrangements of $\{0,0,1,2\}$

4 arr. of $\{0,1,1,1\}$:
0 1 1 1
1 0 1 1
1 1 0 1
1 1 1 0

4 arr. of $\{0,0,0,3\}$:
0 0 0 3
0 0 3 0
0 3 0 0
3 0 0 0

3 In the pay-off matrix below, the entry in each cell is of the form (r, c), where r is the pay-off for the player on rows and c is the pay-off for the player on columns when they play that cell.

	P	Q	R
X	(1, 4)	(5, 3)	(2, 6)
Y	(5, 2)	(1, 3)	(0, 1)
Z	(4, 3)	(3, 1)	(2, 1)

(i) Show that the play-safe strategy for the player on columns is P. **[2]**

(ii) Demonstrate that the game is not stable. **[2]**

The pay-off for the cell in row Y, column P is changed from (5, 2) to (y, p), where y and p are real numbers.

(iii) What is the largest set of values A, so that if $y \in A$ then row Y is dominated by another row? **[1]**

(iv) Explain why column P can never be redundant because of dominance. **[1]**

i.

	P	Q	R
worst pay-off for column	2	1	1

$2 > 1 \therefore$ P is better than Q & R

ii. if column player plays P (safe), row player performs best with Y (1, 5, 4). but then columns player does better by changing from P to Q.

iii. $A = \{y : y \leq 4\}$

iv. in Z, P has the best pay-off for columns player $Z: 3 > 1$

4 The complete bipartite graph $K_{3,4}$ connects the vertices $\{2, 4, 6\}$ to the vertices $\{1, 3, 5, 7\}$.

(i) How many arcs does the graph $K_{3,4}$ have? **[1]**

(ii) Deduce how many different paths are there that pass through each of the vertices once and once only. The direction of travel of the path does not matter. **[3]**

The arcs are weighted with the product of the numbers at the vertices that they join.

(iii) **(a)** Use an appropriate algorithm to find a minimum spanning tree for this network. **[3]**

(b) Give the weight of the minimum spanning tree. **[1]**

i. 12 arcs

ii. Start @ 1, 3, 5, or 7

$4 \text{ choices} \times 3 \times 3 \times 2 \times 2 = 144$ choices

can travel each path in both directions: $\frac{144}{2} = 72$

iii. a) use Kruskal's algorithm:

Weights:

$1 \times 2 = 2$

$1 \times 4 = 4$

$1 \times 6 = 6$

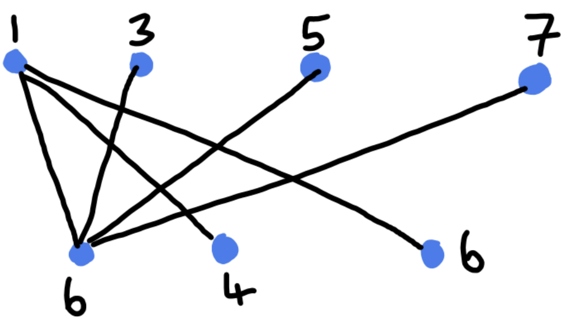
$2 \times 3 = 6$

$2 \times 5 = 10$

~~$3 \times 4 = 12$~~ forms cycle

$2 \times 7 = 14$

etc. (only need 6 arcs)



b) 42

5 Greetings cards are sold in luxury, standard and economy packs.

The table shows the cost of each pack and number of cards of each kind in the pack.

	Pack	Cost (£)	H Handmade cards	F Cards with flowers	A Cards with animals	O Other cards	Total number of cards
LP	Luxury	6.50	10	5	5	0	20
SP	Standard	5.00	5	10	5	10	30
EP	Economy	4.00	0	10	10	20	40

Alice needs 25 cards, of which at least 8 must be handmade cards, at least 8 must be cards with flowers and at least 4 must be cards with animals.

(i) Explain why Alice will need to buy at least two packs of cards. **[2]**

i. 1LP doesn't contain enough cards with flowers.
1SP or 1EP don't contain enough handmade cards.

Alice does not want to spend more than £12 on the cards.

(ii) (a) List the combinations of packs that satisfy all Alice's requirements. **[2]**

(b) Which of these is the cheapest? **[1]**

Ben offers to buy any cards that Alice buys but does not need. He will pay 12 pence for each handmade card and 5 pence for any other card.

Alice does not want her net expenditure (the amount she spends minus the amount that Ben pays her) on the cards to be more than £12.

(iii) Show that Alice could now buy two luxury packs. **[2]**

ii. a) ·1LP + 1SP £11.5 b) 2SP
·1LP + 1EP £10.5
·2SP £10

iii. 2LP meets requirements but costs £13
B buys 40−25 = 15 cards
handmade: 20−8 = 12 cards bought by B
3 remaining cards bought
(12x12p) + (3x5p) = £1.59 ⇒ net cost = £11.41 < £12

Alice decides to buy exactly 2 packs, of which x are luxury packs, y are standard packs and the rest are economy packs.

(iv) Give an expression, in terms of x and y only, for the number of cards of each type that Alice buys. **[4]**

Alice wants to minimise her net expenditure.

(v) Find, and simplify, an expression for Alice's minimum net expenditure in pence, in terms of x and y. You may assume that Alice buys enough cards to satisfy her own requirements. **[3]**

(vi) Find Alice's minimum net expenditure. **[2]**

iv. $H: 10x + 5y$

$F: 5x + 10y + 10\overbrace{(2-x-y)}^{\#EP} = 20 - 5x$

$A: 5x + 5y + 10(2-x-y) = 20 - 5x + 5y$

$O: 10y + 20(2-x-y) = 40 - 20x - 10y$

v. $650x + 500y + 400(2-x-y) + 12(10x + 5y - 8) - 5(10x + 25y + 40(2-x-y) - 17)$

$= 581 + 280x + 115y$

vi. min.: x small as possible, y small as possible & still meeting criteria.

$x = 0, \; y = 2$

$2 \times £5 = £10$. $\overbrace{2(12p) + 33(5p)}^{B \text{ buys}} = £1.89$

$£10 - £1.89 = £8.11$

6 Sheona and Tim are making a short film. The activities involved, their durations and immediate predecessors are given in the table below.

	Activity	Duration (days)	Immediate predecessors	S	T
A	Planning	2	–	✓	✓
B	Write script	1	A	✓	
C	Choose locations	1	A		✓
D	Casting	0.5	A	✓	
E	Rehearsals	2	B, D	✓	
F	Get permissions	1	C		✓
G	First day filming	1	E, F	✓	
H	First day edits	1	G		✓
I	Second day filming	0.5	G	✓	
J	Second day edits	2	H, I		✓
K	Finishing	1	J	✓	✓

(i) By using an activity network, find:

- the minimum project completion time
- the critical activities
- the float on each non-critical activity. **[7]**

(ii) Give two reasons why the filming may take longer than the minimum project completion time. **[2]**

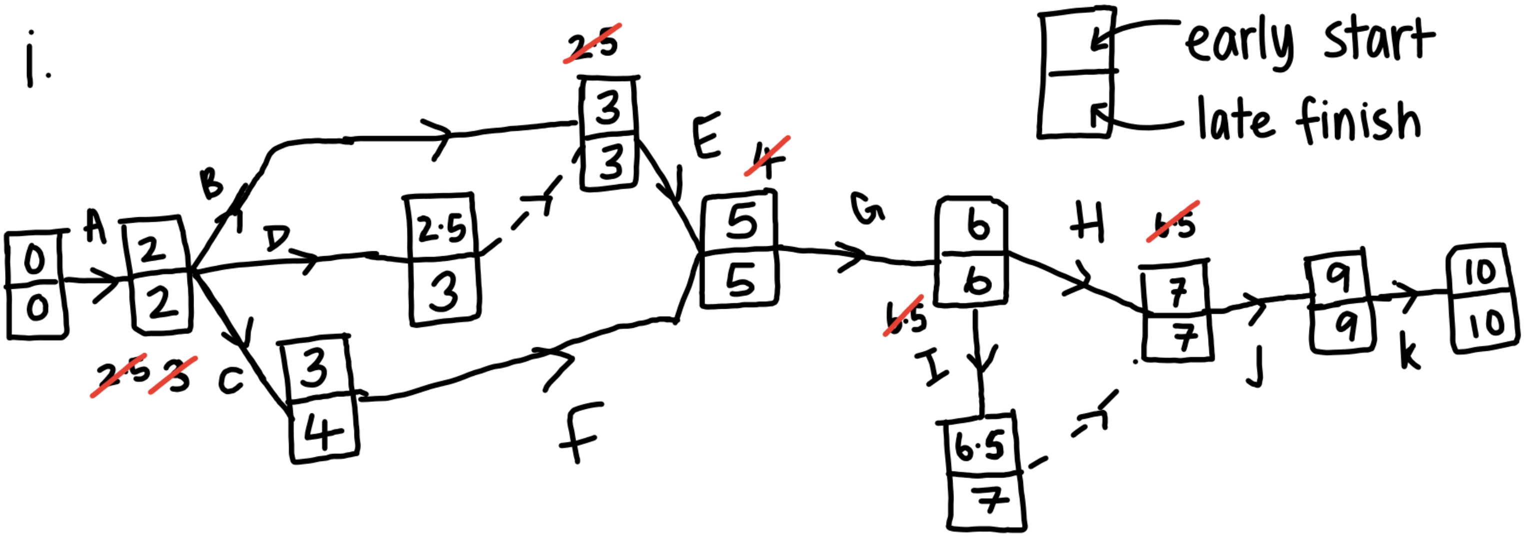
Each activity will involve either Sheona or Tim or both.

- The activities that Sheona will do are ticked in the S column.
- The activities that Tim will do are ticked in the T column.
- They will do the planning and finishing together.
- Some of the activities involve other people as well.

An additional restriction is that Sheona and Tim can each only do one activity at a time.

(iii) Explain why the minimum project completion is longer than in part **(i)** when this additional restriction is taken into account. **[2]**

i.



→ min. completion time: 10 days

critical activities: A, B, E, G, H, J, K

activity	C	D	F	I
float (days)	1	0.5	1	0.5

ii. they might need to wait for special equipment, or a crew member might fall ill, meaning they wouldn't have enough people.

an activity might overrun or be delayed.

iii. E can only start after A, B, & D are done. these can only be done by S, ⇒ can't be done at same time.

duration of ABD: $2+1+0.5=3.5$ days

∴ E delayed until 3.5 (0.5 day delay)

E is a critical activity ∴ delays whole project.

(iv) The project must be completed in 14 days. Find:

(a) the longest break that either Sheona or Tim can take, **[2]**

(b) the longest break that Sheona and Tim can take together, **[2]**

(c) the float on each activity. **[2]**

END OF QUESTION PAPER

iv. a) 8 days each of work

$14 - 8 = 6$ days rest

b) for G: earliest start @ 6, latest @ 10

S busy for 6·5d before end of G, 1·5d after G

T " " 4d " " 4d " "

A→I done as early as possible & J→K as late as possible ⇒ S&T on break from 8·5 to 11 ⇒ 3·5d

c) A, E, G, H, J, K = 3·5d float

B	C	D	F	I
4	5	4·5	5	4